

Algorithms of 3D Cone Beam Tomography For Incomplete Data

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In X-ray computerized tomography, we often have to do with incomplete data. Also often we have situations when we are interested in values of the density not in all points of the object, but only in some domain. The typical example is the control of quality of welded joints of pipes. We need not determine the density in all points of the pipe, but only near by the welded joint. It is desirable also to limit a range of directions of X-rays. In similar situations, the methods of three-dimensional cone beam tomography may be useful.

In classical computerized x-ray tomography, a three-dimensional object usually is being considered as a set of thin cuts, and the two-dimensional Radon inversion formulae are used for reconstruction of the density function. But for analysis of some objects another schema is more natural, when an x-ray source is moving along a three-dimensional curve. Here we have a mathematical problem of reconstruction of a function of three variables from its integrals along the lines that cross the 3D-curve.

Let a function $f(x)=f(x_1,x_2,x_3)$, a vector $\alpha=(\alpha_1,\alpha_2,\alpha_3)$ and a point $z=(z_1,z_2,z_3)$ be given. We define

$$q_f^+(\alpha,z) = \int_0^\infty f(t\alpha + z) dt$$

The function $q_f^+(\alpha,z)$ is the integral of $f(x)$ along the half-line $t \geq 0$ of direction α and passing through the point z .

In [7,11] there are found inversion formulae on base of the Fourier transform (in the distribution sense) of the function $q_f^+(\alpha,z)$. The function $q_f^+(\alpha,z)$ is homogeneous with respect to α , and their Fourier transform does not exist in L_1 . For creating of numerical algorithms, it is necessary to have convergent procedures for all used functions. In [3-5, 8-10] such procedures were given. We will use the inversion formula in

the form of [5,9,10]. If the x-ray source moves along the curve $\Phi(\lambda)=(\Phi_1(\lambda),\Phi_2(\lambda),\Phi_3(\lambda))$, then

$$f(x) = (1/8\pi^2) \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \cos \varphi \frac{1}{(\Phi(\lambda), \beta)} \frac{\partial}{\partial \lambda} \left[\int_{S(\beta)} L(\beta, D) q^+(\gamma, \lambda) d\gamma \right] d\varphi d\theta,$$

where $\beta = \beta(\varphi, \theta) = (\cos\theta \cos\varphi, \sin\theta \cos\varphi, \sin\varphi)$, $S(\beta) = \{\gamma \in S^2 \mid (\beta, \gamma) = 0\}$, S^2 is unit sphere in R^3 ,

$$L(\beta, D) = \sum_{k=1}^3 \beta_k \frac{\partial}{\partial \gamma} k,$$

λ is such that $(\beta, x) = (\beta, \Phi(\lambda))$ and $(\beta, \Phi'(\lambda)) \neq 0$. It is assumed that such λ exists for the point x belonging to the support of the function $f(x)$ (the Kirillov-Tuy conditions are true). From geometric point of view, the Kirillov-Tuy conditions mean that any plane intersecting the point under reconstruction, also intersects (transversally) the trajectory of the source.

For spherical objects the natural example of Kirillov-Tuy trajectory is a union of two circumferences lying in orthogonal planes, for cylindrical objects the natural example of K-T trajectory is a spiral.

It is well known that the Radon transform inversion formula in 2D-space is not local. For calculating the density in the point X , it necessary to have information on all beams intersecting the object, but not only on beams that intersect some neighbourhood of the point X . In contrast with 2D-space, the problem of inversion of the beam transform in the 3D-space is local in some sense. If the Kirillov-Tuy conditions are true in a point X of the object, we can calculate the density in this point although in object there may exist the points, for which the K-T conditions are not true. This fact allows, in particular, for welded joints control to use only a part of the trajectory. For example by

investigation of transverse welded joints of pipes it is possible to use trajectory that have two circles. The first circle lies in the plane $z=0$ and the second circle lies in the plane that is inclined with the angle α (Fig.1).

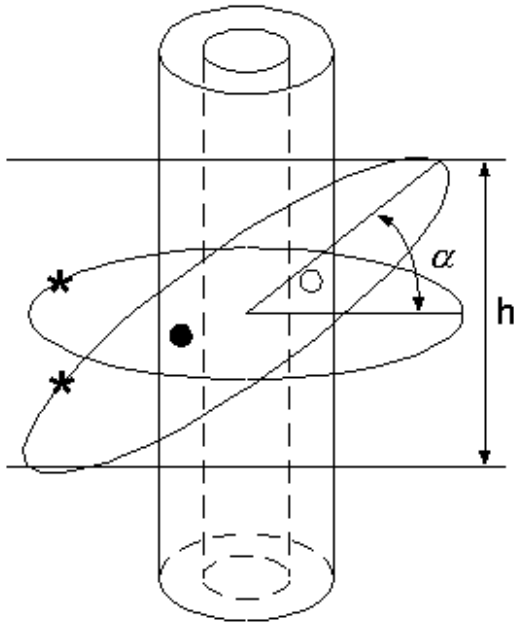


Figure 1

Such a schema allows investigation of an object layer along the axis Z. In the classical computerized tomography, the density in this direction is considered to be constant, and scanning with a step dz is performed. To investigate an object height h , h/dz scans are necessary. The cone-beam tomography reconstruction requires two scans, provided the angle α chosen from the condition $2\cos\alpha=h$.

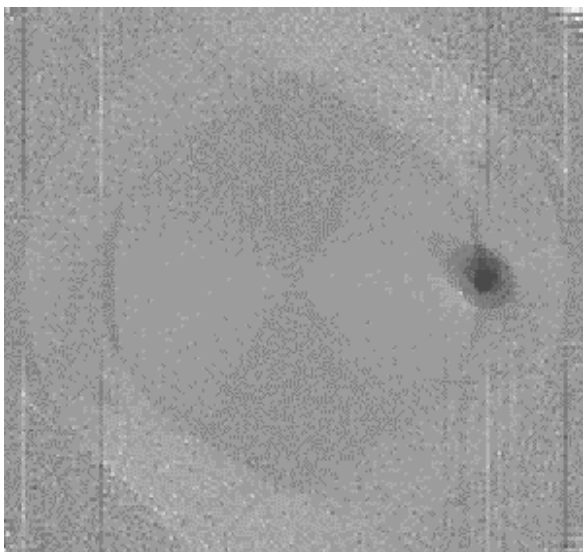


Figure 2

On the Fig.2 are presented the results of computer modeling of reconstruction of the pipe density. The pipe has defects. The pipe density is 4, material in the pipe has density 3, the defect has density zero and is union of four spheres.

In the formula (1) it is need to make integration from zero to 2π , but calculation process allows to give the intermediate results. On the Fig. 3 is presented the result by integration from 0 to π . We can see defects yet on this step. This acceptance allows to shorten time calculation and to use only part of the source trajectory.

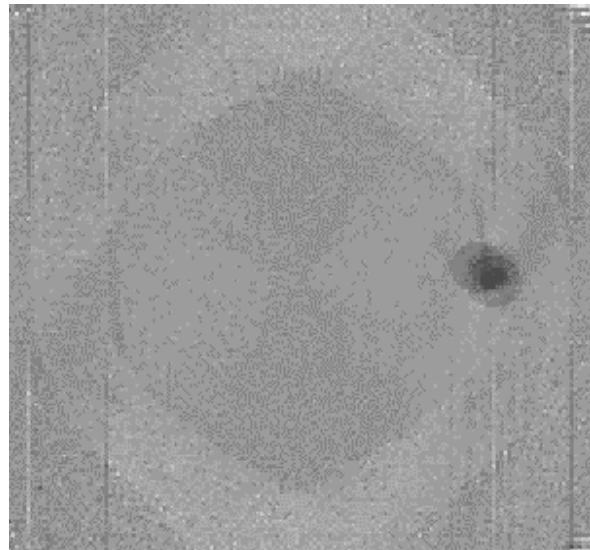


Figure 2

It should be noted that the Kirillov-Tuy conditions are not necessary for cone beam reconstruction [1, 2, 6]. In particular, for reconstruction of the object in 3D space, it is sufficient to have one circumference as trajectory of the source. For this trajectory the reconstruction procedure is unstable with respect to the Sobolev scale [6], but it is possible to select a stable part of this procedure. This part is calculation of integrals along the lines that intersect the circle if integrals along lines intersecting the circumference are given.

On the Fig. 4 is presented example of investigation of object that is analogous the previous object, but the pipe is empty and density of all defects is zero. The source trajectory has only one circle that lies in the plane $z=0$. On base corresponding cone-beam transform were calculated the integral densities along lines that are parallel the z-axis. We can see clear defects in the pipe.

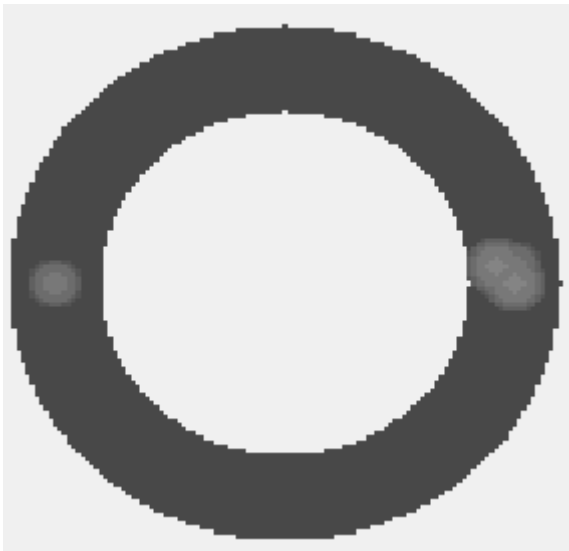


Figure 3

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